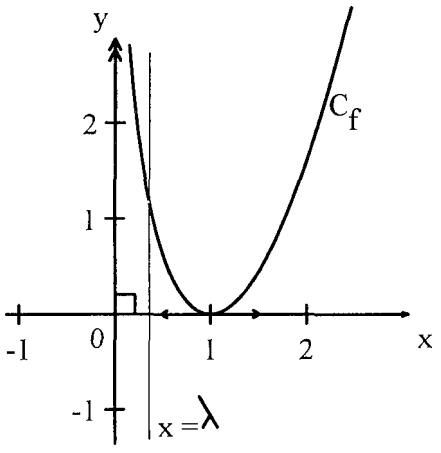
$* \lim\limits_{+\infty} \frac{f(x)}{x} = \lim\limits_{x\to+\infty} \left[ x - 2\frac{Logx}{x} - 1 \right] = +\infty \Rightarrow C_f$ possède une branche parabolique infinie de direction l'axe des ordonnées au voisinage de $+\infty$.




**2)a-** $A(\lambda) = \int_\lambda^1 f(x)dx$

$= \int_\lambda^1 (x^2 - 2Logx - 1)dx$

$= \int_\lambda^1 (x^2 - 1)dx - 2\int_\lambda^1 Logx.\, dx$

$\left( \begin{array}{ll} u'(x)=1 & \curvearrowright \ u(x)=x \\ v(x)=Log(x) & \curvearrowright \ v(x)=\frac{1}{x} \end{array} \right)$

$= \left[ \frac{1}{3}x^3 - x \right]_\lambda^1 - 2\left[ [xLogx]_\lambda^1 - \int_\lambda^1 dx \right]$

$= \frac{1}{3} - 1 - \frac{1}{3}\lambda^3 + \lambda + 2\lambda Log\lambda + 2(1-\lambda) = \frac{4}{3} - \frac{1}{3}\lambda^3 + 2\lambda Log\lambda - \lambda.$

**b-** $\lim\limits_{\lambda\to0^+} A(\lambda) = \lim\limits_{\lambda\to0^+} \left[ \frac{4}{3} - \frac{1}{3}\lambda^3 + 2\lambda Log\lambda - \lambda. \right] = \frac{4}{3}$ car $\lim\limits_{\lambda\to0^+} (\lambda Log\lambda) = 0$

**II]1)** Soit n un entier naturel tel que $n \geq 2$.

**a-** $\cdot 1 \leq k \leq n-1 \Rightarrow 0 < \frac{1}{n} \leq \frac{k}{n} \leq 1 - \frac{1}{n} < 1$

$\cdot 1 \leq k \leq n-1 \Leftrightarrow 2 \leq k+1 \leq n \Leftrightarrow \frac{2}{n} \leq \frac{k+1}{n} \leq 1$

$\Rightarrow 1 \leq k \leq n-1 \Rightarrow \frac{k}{n}$ et $\frac{k+1}{n}$ appartiennent à $]0;1]$.

$\forall t \in [\frac{k}{n}; \frac{k+1}{n}]$ ; $f(\frac{k+1}{n}) \leq f(t) \leq f(\frac{k}{n})$ car f est $\searrow$ sur $]0;1]$

$\Rightarrow \int_{\frac{k}{n}}^{\frac{k+1}{n}} f(\frac{k+1}{n})dt \leq \int_{\frac{k}{n}}^{\frac{k+1}{n}} f(t)dt \leq \int_{\frac{k}{n}}^{\frac{k+1}{n}} f(\frac{k}{n})dt$

$\Leftrightarrow f(\frac{k+1}{n}) \int_{\frac{k}{n}}^{\frac{k+1}{n}} dt \leq \int_{\frac{k}{n}}^{\frac{k+1}{n}} f(t)dt \leq f(\frac{k}{n}) \int_{\frac{k}{n}}^{\frac{k+1}{n}} dt$

$\Leftrightarrow f(\frac{k+1}{n})[\frac{k+1}{n} - \frac{k}{n}] \leq \int_{\frac{k}{n}}^{\frac{k+1}{n}} f(t)dt \leq f(\frac{k}{n})\frac{1}{n}$

$\Leftrightarrow \frac{1}{n}f(\frac{k+1}{n}) \leq \int_{\frac{k}{n}}^{\frac{k+1}{n}} f(t)dt \leq \frac{1}{n}f(\frac{k}{n})$.

**b-** $\forall k \in IN$ tel que $1 \leq k \leq n-1$ on a

$\frac{1}{n}f(\frac{k+1}{n}) \leq \int_{\frac{k}{n}}^{\frac{k+1}{n}} f(t)dt \leq \frac{1}{n}f(\frac{k}{n})$.

$\Rightarrow \sum_{k=1}^{n-1} \frac{1}{n}f(\frac{k+1}{n}) \leq \sum_{k=1}^{n-1} \int_{\frac{k}{n}}^{\frac{k+1}{n}} f(t)dt \leq \sum_{k=1}^{n-1} \frac{1}{n}f\left(\frac{k}{n}\right)$.